Research on a kind of direct method of solving fractional Calculus

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Abstract. By using the direct method, we use an approximate method to solve the fractional order differential equation with the help of Volter equation. We take a triangular function as an example to do the numerical simulation and find that the solution is also like a triangular function. And we should point out that the disadvantage of this direct method is that each step should do a integration calculation so it need a lot time to finish the simulation if the simulation time is very long and the simulation step is set very small.

Introduction

Fractional Calculus is the theory of studying arbitrary order differential and integral, and it is the generalization and extension of traditional integer order calculus theory. It has a history of more than 300 years [1-6]. Fractional-Order Systems are systems described by differential equations with arbitrary real order. In recent years, Fractional-order Control systems, which are controlled by fractional-order systems or fractional-order controllers, have gradually become a new research field in the control field [7-14]. At present, there are many theoretical studies on fractional differential equation or system, some of which are focused on its stability and some on its approximate simulation principle [15-21]. In the aspect of approximate numerical simulation, there are many methods, which are well known as fractional order simulation based on prediction method; secondly, approximate approximation of fractional order by transfer function approximation method. However, there are few studies on integration simulation using direct method based on fractional order definition, mainly because of the complexity of this kind of direct method. According to the definition of fractional order, this kind of direct method is simulated and analyzed in this paper.

Definition of Fractional Order Differential

According to the fractional definition, a fractional system can be described as

$$\frac{d^q x}{dt^q} = f(t,x) \tag{1}$$

And its Volter equation can be written as

$$x(t) = \sum_{k=0}^{m-1} x_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau, x(\tau)) d\tau \tag{2}$$

Where $x^{(k)}(0) = x_0^{(k)}$, $k = 0,1,\cdots,m-1$, $q \in (m-1,m)$.

Setting of Simulation of Fractional Order Differential

Choose a function as follows as an example to do the fractional order simulation:

$$f(t, x)=\sin(t) \tag{3}$$

Then

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$$f(\tau, x(\tau)) = \sin(t) \tag{4}$$

And choose the fractional order as q=1/5, then m=1, and set the initial value as x(0)=1. Set the simulation step as 0.1s, and the simulation time is set as 5s, then it holds

$$x_0^{(k)} \frac{t^k}{k!} = x_0^{(0)} = x_0 \tag{5}$$

Where

$$\frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau, x(\tau)) d\tau = \frac{1}{\Gamma(1)} \int_0^t (t-\tau) f(\tau, x(\tau)) d\tau \tag{6}$$

And it can be rewritten as

$$f(\tau, x(\tau)) = \sin(\tau) \tag{7}$$

If $t = 0.3$, then it can be written as

$$x(0.3) = x_0 + \frac{1}{\Gamma(1)} \int_0^{0.3} (0.3-\tau) \sin(\tau) d\tau \tag{8}$$

So there is an integration calculation in each step of fractional order simulation, which is why it is very complex to use the direct method.

Simulation Program

We can write a short simulation program by using the M language with Matlab software according to above analysis.

```
clc;clear;close all;
tf=5;dt=0.1; x0=1;q=1/5;
  x=x0;
for i=1:tf/dt
        t=i*dt;
          tp(i)=t;
        xp(i)=x;
        se=0;
        for j=1:i
                tao=j*dt-1;
                xtao=sin(tao);
                ftaoxtao=xtao;
                dse=(t-tao)^(q-1)*ftaoxtao;
                se=se+dse*dt;
                sep(i,j)=se;
            end
        x=x0+gamma(q)*se;
end
plot(tp,xp,'*')
```

Simulation Result

With above program, and we get the simulation result of above fractional order differential equation by using the direct method.

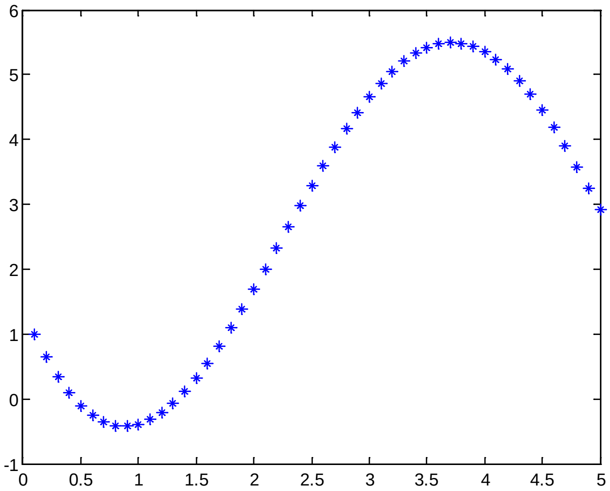

Figure 1 The state of fractional system

And since the simulation step is 0.1, so the calculation is not very big. If we decrease the simulation step and increase the simulation time, then the calculation for computer to finish this simulation will be greatly increased.

Result Analysis

From the above simulation images, it can be seen that the result of fractional integration of sinusoidal function is still like a triangular function. And the amplitude increases, which is similar to integer differential. The advantage of the whole fractional-order direct simulation method lies in its simple logic, but its disadvantage is that it can not be accumulated. With the increase of simulation time, the amount of calculation increases in geometric series. Dimensional direct simulation requires very high computer memory.

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